Exact solution for an anisotropic star admitting the MIT Bag model equation of state

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 Abstract

Current observational data indicates the possible existence of compact stars composed of deconfined quarks known as strange stars. In this paper, we present two new class of solutions for a compact star whose interior strange matter composition admits the MIT Bag model equation of state. The solutions are expressed in terms of elementary functions which facilitates its physical applicability. The new solution contains some of the previously obtained solutions including models of charged anisotropic relativistic spheres and isotropic charged star model. The general class of solutions generated in this paper will, hopefully, contribute to the existing rich class of solutions to the Einstein-Maxwell system of equations.

Keywords: Compact stars, Einstein-Maxwell system, Exact solutions, MIT bag model EOS.

1. Introduction

Theoretical modelling of quark stars has received intense research interests ever since quarks have been conjectured to be the most energetically favourable state of baryonic matter Witten [1], Farhi and Jaffe [2]. In a seminal work, the existence of a stellar configuration in hydrostatics equilibrium composed of quark matter was first proposed by Itoth [3]. Since we do not observe free quarks, in an attempt to describe the quark confinement mechanism, [4] proposed the phenomenological MIT Bag model where one assumes that the quark confinement is caused by a universal pressure called the 'bag pressure' at the boundary of the region containing quarks. The equation of state (EOS) in the bag model has a simple linear form $p = (\rho - 4B)/3$, where ρ = density, p = isotropic pressure and B = the bag constant. For stability, one assumes the bag constant within a particular range [2, 5], even though density and temperature dependent bag models have been proposed where the bag constant can take a much wider range of values [6, 7]. Stellar configurations entirely composed of deconfined u, d and s quarks are so compact systems that a relativistic approach is necessary to describe such objects. Consequently, relativistic stellar models composed entirely of quark matter have been developed and analyzed to understand the physical properties of such type of stars. The analytic models have been developed either by choosing the simplest linear form of the bag model EOS or a more complex EOS for quark matter. For some recent treatments in this area, one may follow the works of [8, 9, 10] and references therein.

The aim of the current investigation is to present new class of exact solutions capable of describing strange star candidates. We intend to achieve this by generating a new class of exact solutions to the Einstein-Maxwell system for a spherical object composed of quark matter in the presence of

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local anisotropy. For the quark matter, we assume the bag model equation of state and *to* solve the system of equations, we assume a particular form of one of the metric variables and also the radial fall-off behaviour of the anisotropic parameter that has been used in a recent paper by Maharaj et al [11]. The advantage of this approach is that one can regain the charged isotropic stellar model simply by setting the anisotropy to zero. It is interesting to note that many previously found explicit solutions of the Einstein-Maxwell system with anisotropic stress e.g., solutions obtained by [10, 12, 13, 14, 15, 16] do not have their corresponding isotropic analogues.

It is now a well-established fact that local anisotropy plays a significant role in the studies of relativistic stellar objects [17, 18, 19]. In the case of strange stars composed of u, d and s quarks, the role of anisotropic stress has been analyzed by [20, 21, 22]. Physically significant parameters like surface redshift, luminosity and the maximum mass of a compact star do get significantly influenced by the presence of electric field and anisotropic stress have been shown in [22, 23]. Since quark stars are expected to possess a huge electromagnetic field Usov [24], it is quite natural to study stellar configurations by incorporating electromagnetic field as well. Consequently, a wide variety of charged stellar models have been developed and studied which are available in the compilation work of Ivanov [25].

Incorporation of electromagnetic field and anisotropy makes the system of field equations even more difficult to solve unless one adopts some simplifying techniques to make them tractable. In an earlier work, by identifying a conformal Killing vector, Mak and Harko [26] developed a relativistic model of an isotropic quark star. The work was later extended by Komathiraj and Maharaj [27] who provided a more general class of exact solutions by incorporating an electromagnetic field in the system of field equations. In a more recent work, Maharaj et al [11] have made a further generalization of [27] model by incorporating anisotropic stress into the system. In a subsequent paper, Sunzu et al [28] performed a detailed physical analysis of the solution obtained in [11] and discussed its relevance in the context of compact quark stars candidates. It is interesting to note that the class of solutions generated in [11] for an assumed form of the anisotropic parameter $\Delta = A_0 + A_1 x + A_2 x^2 + A_3 x^3$ can be reduced to the charged isotropic stellar solutions of [26] and [27]. In this work, we choose a different form of the measure of anisotropy which, interestingly, provides much simpler analytic solutions.

The paper has been organized as follows: In Section 2, making use of the Durgapal and Bannerji [29] transformations, we have laid down an equivalent set of differential equations of the Einstein-Maxwell field equations for a matter distribution which admits a bag model EOS. To solve the system, we have presented the basic assumptions in Section 3. Two new class of solutions, in terms of elementary functions, have been obtained in Section 4. The charged isotropic solutions found earlier in [26, 27, 30] have been shown to be special cases of our general class of solutions. In Section 5, we have analyzed regularity, physical requirements and subsequent bounds of our model. For an assumed set of model parameters, consistent with the bounds, physical acceptability of a particular class of solution has been shown. We have concluded by discussing our main results in Section 6.

2. Einstein-Maxwell system of equations

We write the line element of a spherically symmetric relativistic fluid sphere in coordinates $(x^a) = (t, r, \theta, \phi)$ as

$$ds^2 = -e^{2\mu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),\tag{1}$$

where $\mu(r)$ and $\lambda(r)$ are yet to be determined. The Einstein-Maxwell system of field equations corresponding to the line element (1), are obtained as

$$\frac{1}{r^2}(1 - e^{-2\lambda}) + \frac{2\lambda'}{r}e^{-2\lambda} = \rho + \frac{1}{2}E^2,\tag{2a}$$

$$-\frac{1}{r^2}(1 - e^{-2\lambda}) + \frac{2\mu'}{r}e^{-2\lambda} = p_r - \frac{1}{2}E^2,\tag{2b}$$

$$e^{-2\lambda}\left(\mu'' + \mu'^2 + \frac{\mu'}{r} - \mu'\lambda' - \frac{\lambda'}{r}\right) = p_t + \frac{1}{2}E^2,\tag{2c}$$

$$\frac{1}{r^2}e^{-\lambda}(r^2E)' = \sigma,\tag{2d}$$

In Equations (2), E and σ denote the electric field intensity and the proper charge density respectively and a prime (') denotes derivative with respect to the radial coordinate r. In these equations and hereafter, we have used units where $8\pi G = c = 1$. The energy density ρ, radial pressure p_r and the tangential pressure p_t are measured relative to the comoving fluid 4-velocity $u^a = e^{-\mu}\delta_0^a$.

We assume that the interior of the star is composed entirely of u, d and s quarks and accordingly, we use the MIT bag model EOS

$$p_r = \frac{1}{3}(\rho - 4B).\tag{3}$$

Now by introducing the Durgapal and Bannerji [29] transformations

$$A^2y^2(x) = e^{2\mu(r)}, \quad Z(x) = e^{-2\lambda(r)}, \quad x = Cr^2,\tag{4}$$

the system of equations (2) can be obtained in the following form

$$\rho = 3p_r + 4B,\tag{5a}$$

$$\frac{p_r}{C} = Z\frac{\dot{y}}{y} - \frac{\dot{Z}}{2} - \frac{B}{C},\tag{5b}$$

$$p_t = p_r + \Delta,\tag{5c}$$

$$\frac{\Delta}{C} = \frac{4xZ\ddot{y}}{y} + (6Z + 2x\dot{Z})\frac{\dot{y}}{y} + \left[2\left(\dot{Z} + \frac{B}{C}\right) + \frac{Z-1}{x}\right],\tag{5d}$$

$$\frac{E^2}{2C} = \frac{1-Z}{x} - 3Z\frac{\dot{y}}{y} - \frac{\dot{Z}}{2} - \frac{B}{C},\tag{5e}$$

$$\sigma = 2\sqrt{\frac{CZ}{x}}(E + x\dot{E}).\tag{5f}$$

where dot (.) denotes differentiation with respect to the variable x; $\Delta = p_t - p_r$ represents the measure of anisotropy and A and C are arbitrary constants. The mass of a self-gravitating object for a given radius is an important measure for comparison with observational data. In this case, the mass contained within a radius x of the sphere is obtained as

$$m(x) = \frac{1}{4C^{\frac{3}{2}}}\int_0^x \sqrt{x}\rho(x)dx.\tag{6}$$

3. Scheme for generating new solutions

To integrate the system (5), we need to specify any two of the following variables: $y, Z, \rho, p_r, p_t, \Delta, E$ or σ. We solve the Einstein-Maxwell system by choosing a specific form of the gravitational potential y and the anisotropic parameter Δ. Once the gravitational potential y and the anisotropic parameter Δ are specified, the gravitational potential Z can be found by integrating (5d) which is a first order linear equation in Z. The remaining unknowns p_r, ρ, p_t and E^2 are then obtained from (5b), (5a), (5c) and (5e), respectively. Thus, we have a complete solution of the Einstein-Maxwell system of equations. We shall follow this approach in this paper.

We first choose the metric function in the form

$$y(x) = (a + bx^{1-m} + x^m)^n, \tag{7}$$

where a, b, m and n are constants. The choice (7) ensures that the metric function is regular at the centre and is well behaved within the stellar interior. Moreover, this particular choice contains some special cases of previously developed models namely, the case (i) $a = b = 0, m = \frac{1}{2}$ and $n = 1$ corresponds to [25] model; and the case (ii) $(b = 1, m = 0)$ or $(b = 0, m = 1)$ with $n = 2$ corresponds to [27] model of quark stars with isotropic pressure and [11] model with anisotropic stresses. In this paper, we intend to study the Einstein-Maxwell system in the presence of an anisotropy stress.

Substitution of equation (7) in (5d) yields a first order differential equation

$$\dot{Z} + \left[\frac{1}{2x} + \frac{2(n-1)(b(1-m) + mx^{2m-1})}{x(b + ax^{m-1} + x^{2m-1})} + \frac{b(1-m)\gamma + m\varphi x^{2m-1}}{2x(b\omega + ax^{m-1} + (1+mn)x^{2m-1})} \right] Z$$

$$- \frac{\left(1 - \frac{2Bx}{C}\right)(b + ax^{m-1} + x^{2m-1})}{2x(b\omega + ax^{m-1} + (1+mn)x^{2m-1})} = \frac{\Delta(b + ax^{m-1} + x^{2m-1})}{2C(b\omega + ax^{m-1} + (1+mn)x^{2m-1})}, \tag{8}$$

for the metric function Z, where we have set $\gamma = 4 + n - 4nm$, $\varphi = 4 - 3n + 4nm$ and $\omega = 1 + n - nm$. To integrate (8), we choose the anisotropy in the form

$$\Delta = \frac{2\alpha C}{b + ax^{m-1} + x^{2m-1}}, \tag{9}$$

where α is an arbitrary constant. This particular form ensures the regular behaviour of the anisotropy within the stellar interior i.e., it is zero at the centre. Most importantly, this particular choice provides an exact solution with desirable physical features.

Substitution of (9) into (8) yields

$$\dot{Z} + \left[\frac{1}{2x} + \frac{2(n-1)(b(1-m) + mx^{2m-1})}{x(b + ax^{m-1} + x^{2m-1})} + \frac{b(1-m)\gamma + m\varphi x^{2m-1}}{2x(b\omega + ax^{m-1} + (1+mn)x^{2m-1})} \right] Z$$

$$- \frac{\left(1 - \frac{2Bx}{C}\right)(b + ax^{m-1} + x^{2m-1})}{2x(b\omega + ax^{m-1} + (1+mn)x^{2m-1})} - \frac{\alpha}{(b\omega + ax^{m-1} + (1+mn)x^{2m-1})} = 0, \tag{10}$$

Equation (10) can be integrated in terms of elementary functions for specific values of the model parameters as discussed in the following.

3.1 The case $m = \frac{1}{2}$ and $n = 1$

In this case, the solution of equation (10) is obtained in the form

$$Z = \frac{3(2a + d\sqrt{x}) - \frac{Bx}{C}(4a + 3d\sqrt{x}) + 3\alpha x^{3/2}}{3(2a + 3d\sqrt{x})}$$

where, $b + 1 = d$. Consequently, we generate an exact analytical model for the system as:

$$e^{2\mu} = A^2(a + d\sqrt{x})^2, \tag{11a}$$

$$e^{2\lambda} = \frac{3(2a + 3d\sqrt{x})}{3(2a + d\sqrt{x}) - \frac{Bx}{C}(4a + 3d\sqrt{x}) + 3\alpha x^{3/2}}, \tag{11b}$$

$$\rho = \frac{3C(6a^2d + 10ad^2\sqrt{x} + 3d^3x)}{2\sqrt{x}(a + d\sqrt{x})(2a + 3d\sqrt{x})^2} + \frac{B\left(16a^3 + 47a^2d\sqrt{x} + 48ad^2x + 18d^3x^{\frac{3}{2}}\right)}{2(a + d\sqrt{x})(2a + 3d\sqrt{x})^2}$$
$$- \frac{3\alpha\sqrt{x}(3a^2 + 4ad\sqrt{x})}{2(a + d\sqrt{x})(2a + 3d\sqrt{x})^2}, \tag{11c}$$

$$p_r = \frac{\rho - 4B}{3}, \tag{11d}$$

$$p_t = \frac{\rho - 4B}{3} + \frac{2\alpha C\sqrt{x}}{a + d\sqrt{x}}, \tag{11e}$$

$$\Delta = \frac{2\alpha C\sqrt{x}}{a + d\sqrt{x}}, \tag{11f}$$

$$E^2 = \frac{C(-2a^2d - 2ad^2\sqrt{x} + 3d^3x) + Bx(a^2d + 2ad^2\sqrt{x})}{\sqrt{x}(a + d\sqrt{x})(2a + 3d\sqrt{x})^2}$$
$$- \frac{\alpha\sqrt{x}(7a^2 + 22ad\sqrt{x} + 18d^2x)}{(a + d\sqrt{x})(2a + 3d\sqrt{x})^2}. \tag{11g}$$

Interestingly, by setting $\alpha = 0$ and $d = 1(b = 0)$, we regain the 1st class of charged isotropic solutions of [27]. If we set $a = 0$, we obtain

$$e^{2\mu} = A^2Cr^2, \;\; e^{2\lambda} = \frac{3}{1 - Br^2}, \;\; \rho = \frac{1}{2r^2} + B, \;\; p_r = p_t = \frac{1}{6r^2} - B, \;\; E^2 = \frac{1}{3r^2}. \tag{12}$$

which is the quark stellar model of [26]. By setting B = 0 in (12), we regain the [30] solution. However, even though the gravitational potentials remain well behaved for the obtained class of solutions as in previously found solutions of [26, 27, 11]; the matter variables and the electric field suffer from singularity in this case.

3.2 The case $m = 0$ and $n = 2$
In this case, by integrating equation (10), we obtain

$$Z = \Big[9(35d^3 + 35d^2bx + 21db^2x^2 + 5b^3x^3)$$
$$- \frac{2Bx}{C}(105d^3 + 189d^2bx + 135db^2x^2 + 35b^3x^3) + 2\alpha x^2(63d^2 + 90dbx$$
$$+ 35b^2x^2)\Big] \times \frac{1}{315(d + bx)^2(d + 3bx)}$$

where, $a + 1 = d$. The subsequent solution and matter variables are given as:

$$e^{2\mu} = A^2(d + bx)^4, \tag{13a}$$

$$e^{2\lambda} = \frac{315(d + bx)^2(d + 3bx)}{9(35d^3 + 35d^2bx + 21db^2x^2 + 5b^3x^3 - l(x))}, \tag{13b}$$

$$\rho = \frac{6C(70d^4b + 217d^3b^2x + 159d^2b^3x^2 + 75db^4x^3 + 15b^5x^4)}{315(d + bx)^3(d + 3bx)^2}$$
$$+ \frac{2B[3(35d^5 + 133d^4bx + 246d^3b^2x^2) + 5(254d^2b^3x^3 + 209db^4x^4 + 63b^5x^5)]}{105(d + bx)^3(d + 3bx)^2}$$
$$- \frac{\alpha x(126d^4 + 207d^3bx - 535d^2b^2x^2 - 835db^3x^3 - 315b^4x^4)}{105(d + bx)^3(d + 3bx)^2}, \tag{13c}$$

$$p_r = \frac{\rho - 4B}{3}, \tag{13d}$$

$$p_t = \frac{\rho - 4B}{3} + \frac{2\alpha Cx}{d + bx}, \tag{13e}$$

$$\Delta = \frac{2\alpha Cx}{d + bx}, \tag{13f}$$

$$E^2 = \frac{Cx(196d^3b^2 + 1452d^2b^3x + 1356db^4x^2 + 420b^5x^3)}{35(d + bx)^3(d + 3bx)^2}$$
$$- \frac{Bx\left(56d^4b + 432d^3b^2x + 2176d^2b^3x^2 + \frac{7280}{3}db^4x^3 + 840b^5x^4\right)}{105(d + bx)^3(d + 3bx)^2}$$
$$- \frac{2\alpha x\left(84d^4 + 633d^3b + 1699d^2b^2x^2 + \frac{4865}{3}db^3x^3 + 525b^4x^4\right)}{105(d + bx)^3(d + 3bx)^2}. \tag{13g}$$

It is to be stressed here that this particular solution is a generalization of the second class of solutions obtained earlier by Komathiraj and Maharaj [27] which can be regained by setting $\alpha = 0$ and $b = 1$. Most importantly, the gravitational potentials and the physical variables are regular and well behaved for this class of solutions which facilitates its applicability for the description of compact stars.

The form of E^2, in this case, is physically palatable as it remains regular and continuous throughout the sphere. In addition, the field intensity E vanishes at the stellar centre ($r = 0$) and remains positive throughout in the interior of the star for appropriate choices of the model parameters.

4. Matching conditions and physical requirements:
By utilizing the matching conditions, regularity conditions and other physical requirements [31], let us now find the appropriate bounds on the model parameters for the particular solution (13):

C1. The gravitational potentials $e^{2\lambda}$ and $e^{2\mu}$ should remain positive throughout the stellar interior. From equations (13a) and (13b), we note that $e^{2\mu}(r = 0) = A^2 d^4, (e^{2\mu})'(r = 0) = 0$ and $e^{2\lambda}(r = 0) = 1, (e^{2\lambda})'(r = 0) = 0$. The results show that the gravitational potentials are regular at the centre $r = 0$.

C2. The energy density and pressure should be non-negative inside the stellar interior. From (13c), we obtain the central density $\rho_0 = \rho(r = 0) = \frac{12bC}{d} + 2B$. Using (13d), we have $p_r(r = 0) = p_t(r = 0) = \frac{4bC}{d} - \frac{2B}{3}$. These results imply that the energy density and the two pressures will be non-negative at the centre if the following condition is satisfied: $\frac{bC}{d} > \frac{B}{6}$.

C3. The interior metric should be matched to the exterior Reissner-Nordstrom metric at the boundary of the star $r = R$. Using this condition, the constant A is obtained in terms of model parameters and the boundary radius.

C4. The requirement $p_r(r = 0) = 0$ yields the bag constant B in terms of model parameters and the boundary radius.

C5. For a realistic star, it is expected that the gradient of density, radial pressure and the tangential pressure should be decreasing functions of the radial parameter r i.e., $\frac{d\rho}{dr} \leq 0, \frac{dp_r}{dr} \leq 0$ and $\frac{dp_t}{dr} \leq 0$. Using equations (3) and (13c) - (13f), this nature has been shown graphically in Fig. 6.

C6. The causality condition demands that the radial and the tangential sound speeds should not exceed the speed of light i.e., $0 < \frac{dp_r}{d\rho} < 1, \ 0 < \frac{dp_t}{d\rho} < 1$. In this model we have $0 < \frac{dp_r}{d\rho} < \frac{1}{3}$. By choosing the model parameters appropriately, we have shown graphically in Fig. 7 that the requirement $0 < \frac{dp_t}{d\rho} < 1$ is also fulfilled in this model.

C7. For a realistic model, the following energy conditions are to be satisfied: (i) The Weak Energy Condition (WEC) $\rho - p_r \geq 0$ and $\rho - p_t \geq 0$. (ii) The Strong Energy Condition (SEC) $\rho - 3p_r \geq 0$ and $\rho - 3p_t \geq 0$. (iii) The Trace Energy Condition (TEC) $\rho - p_r - 2p_t \geq 0$. Since ρ, p_r and p_t are non-negative quantities, the energy condition(s) are satisfied in this model.

C8. For a stable configuration, it is expected that the adiabatic index $\Gamma = \frac{\rho + p_r}{p_r} \frac{dp_r}{d\rho}$, should be greater than 4/3 [32, 33, 34]. The above requirement is fulfilled in our model as can be seen from equation (3).

5. Physical analysis

We have proved that the second class of solution (13) obtained in this paper is regular and well-behaved. Since the solution has been obtained by assuming the bag model EOS for a quark star, one can use the solution to model compact stellar objects like Her X-1 and SAX and J1808.4-3658, among others, which have been claimed to be good strange star candidates in the recent past.

Note that the model contains five constants namely, a, b, C, B and α. The constants a and b appear in the potential y given in equation (7); the constant C has been utilized in the transformation given in (4); B is the bag constant given in equation (3) and α corresponds to the anisotropic factor given in (9). The remaining parameters can be expressed in terms of these constants as $d = a + 1, \ D = bC$. Three of these parameters do get fixed by the matching conditions at the boundary, namely matching of the interior solution to the Schwarzschild exterior metric at $r = R$ and imposition of the requirement that pressure must vanish at the boundary i.e., $p_r(r = R) = 0$. The parameters α

fixes the extent of anisotropy ($\alpha = 0$ implies isotropic configuration). Thus, for a given bag constant within the stability window we have a physically reasonable and well-behaved model.

For a given set of values of the model parameters ($C = 1, b = 0.4, d = 2, B = 0.303422, \alpha = 0.1$ and $A = 0.112372$), which are consistent with the constraints discussed in section 4, we have shown graphically behaviour of the quantities of physical interest in Fig. 1 -7. For numerical calculations, we have used the software package Mathematica [35]. Fig. 1 shows that the gravitational potentials $e^{2\mu}$ and $e^{2\lambda}$ are continuous, regular and well-behaved within the stellar interior. In Fig. 2, we note that that the energy density ρ is positive, finite and decreases radially outward. In Fig. 3, the fall-off bahaviour of the radial pressure p_r tangential pressure p_t and $\rho - p_r - 2p_t$ has been shown. Since the energy density ρ and the radial pressure p_r are linked by a linear equation of state, p_r has the same fall-off behaviour as that of the density. The tangential pressure p_t also decreases radially outward. Obviously, the radial p_r vanishes at the boundary. The tangential pressure need not vanish at the boundary. At $r = 0, p_r = p_t$ which is a desirable feature of a realistic stellar model [24]. The solution satisfies the energy conditions and in Fig. 3 we note that the trace energy condition (TEC) $\rho - p_r - 2p_t \geq 0$ is also satisfied in this model which is a much stronger condition as compared to the other energy conditions [36]. The anisotropic factor Δ, as shown in Fig. 4, is zero at the centre and it monotonically increases until it attains a maximum value at the boundary of the stellar object. This particular behaviour is similar to the results obtained by earlier by [37]. The electric field E^2 is also regular at the cenre; it initially increases and then decreases after reaching a maximum which is similar to many results found earlier including the recent treatment of Komathiraj and sharma [38]. The work of [38] was, in fact, a generalization of the models provided by [29, 39, 40]. In Fig. 5, we note that the mass function $m(r)$ is regular and well behaved. Fig. 6 shows the gradient of density $\frac{d\rho}{dr}$, radial pressure $\frac{dp_r}{dr}$ and tangential pressure $\frac{dp_t}{dr}$ are all negative throughout the star. Fig. 7 shows that $\frac{dp_r}{d\rho}$ and $\frac{dp_t}{d\rho}$ also remain within the desired range [0, 1].
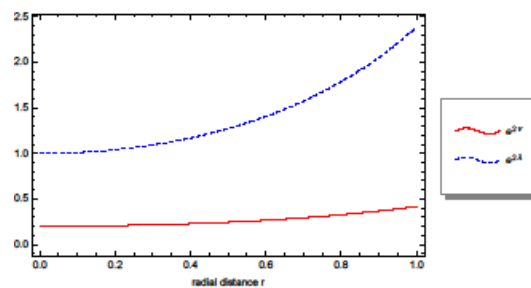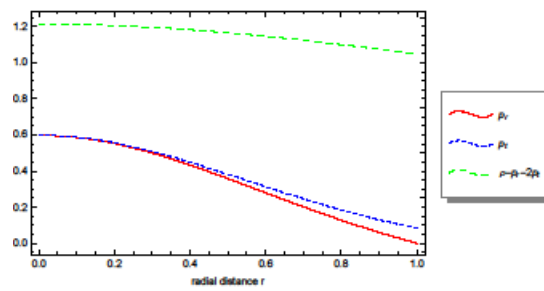

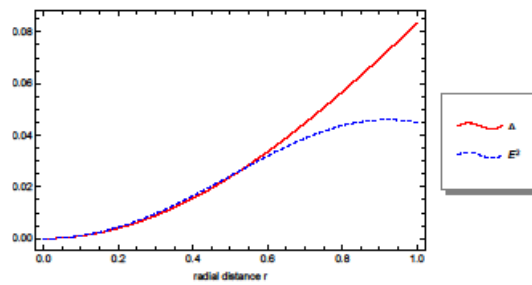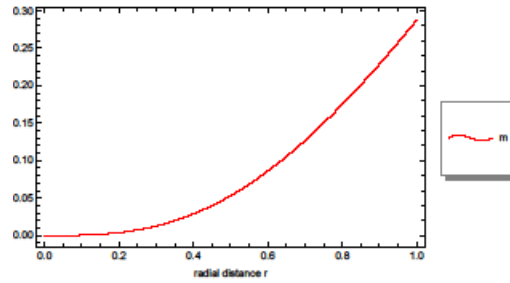**Fig. 1**. Behaviour of gravitational potentials $e^{2\mu}$ *and* $e^{2\lambda}$ at the stellar interior.
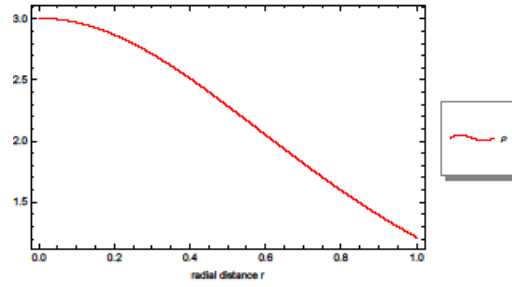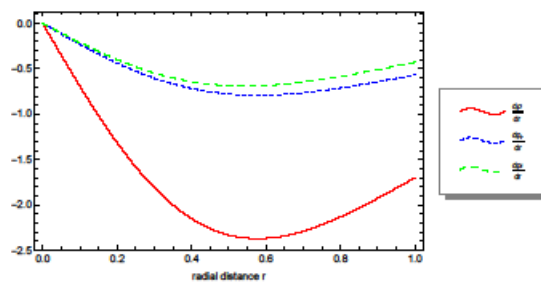
Fig. 2. Radial variation of energy density.

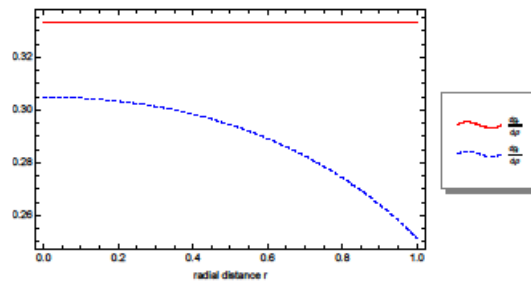**Fig. 3.** Fall-off bahaviour of the radial pressure p_r, tangential pressure p_t and $\rho - p_r - 2p_t$

Fig. 4. Radial variation of anisotropic factor Δ and electric field E^2.

Fig. 5. Mass function $m(r)$ plotted against the radial distance.

Fig. 6. Gradient of density $\frac{d\rho}{dr}$, radial pressure $\frac{dp_r}{dr}$ and tangential pressure $\frac{dp_t}{dr}$.

Fig. 7. Radial variation of $\frac{dp_r}{d\rho}$ and $\frac{dp_t}{d\rho}$.

6. Discussion

To summarize, in this work, we have been able to provide a couple of new solutions for an anisotropic stellar configuration couched on the Reissner-Nordström¨ background spacetime. We have demonstrated that there exist particular values of the model parameters for which a particular class of solution (13) satisfies the requirements of a physically reasonable stellar model. Since the solution has been obtained for a composition admitting a bag model EOS, the solution might be

useful for the description of compact strange star candidates. Hopefully, our results will contribute to the rich class of exact solutions to the Einstein-Maxwell system of field equations. It is to be stressed that we have been able to generate solutions for parameter values (i) $m = \frac{1}{2}$ and $n = 1$; and (ii) $m = 0$ and $n = 2$, only. It will be interesting to check what other values of the model parameters can yield solutions which are regular, well behaved and can describe realistic stars. Such possibilities, however, will be taken up elsewhere.

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