EXPONENTIALLY WEIGHTED MOVING AVERAGE DISTANCE SQUARE SCHEME FOR JOINT MONITORING OF PROCESS MEAN AND VARIANCE

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ABSTRACT : *In quality control, joint monitoring of process mean and variance has become more popular due to the disadvantages of monitoring mean and variance alone. This paper introduce a new joint monitoring scheme for process mean and variance. In this new scheme, exponential moving average technique is applied to the statistics* D_t^2 *developed in the Shewhart distance scheme by Razmy (2010). The optimal design parameter were found through simulations for designing this new scheme. The design techniques of this scheme are illustrated with an example. A big advantage of this new scheme is, unlike most existing joint monitoring schemes, the design parameters of this scheme are independent on the sample size thus the quality engineers have fewer constraints in implementing this scheme..*

Keywords: Average run length, Control limit, Exponential weighted moving average, Joint monitoring,

1. INTRODUCTION

In quality control, joint monitoring (JM) of process mean and variance has emerged to a popular topic after Gan (1997) showing the adverse issues in monitoring the mean and variance alone. Many authors combined the mean and variance charts for JM of process mean and variance under the Shewhart, Exponentially weighted moving average (EWMA) and Cumulative sum schemes**.** The Shewhart control charts developed by Shwhart (1939) for monitoring mean and variance were combined by Gan (1997), Chen and Cheng (1998) and Razmy (2010), and presented different Shewhart JM schemes. These new Shewhart JM schemes were evaluated by McCracken *et. al.* (2013) for quick detection of the shifts in mean and variance and reported that the schemes proposed by Razmy (2010) outperform the other JM schemes. The EWAM chart developed by Roberts (1959) was combined by Gan (1995) and Chen et al (2001, 2004), and presented different EWMA JM schemes. In this paper the concept of Shewhart distance (SD) scheme developed by Razmy (2010) is extended to the EWMA scheme to present a new JM scheme called EWMA distance square schemes (EWMAD2 scheme).

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2. DEVELOPMENT OF NEW SCHEME

The SD scheme is set up by plotting D_t^2 against the sample number t where,

$$D_t^2 = U_t^2 + V_t^2 \qquad [1]$$

The statistics U_t is standardized variable for mean which is given by

$$U_t = \frac{\bar{X}_t - \mu_0}{\sigma_0/\sqrt{n}} \qquad [2]$$

where , $\bar{X}_t$ is the t^{th} sample mean for a process where

$$\bar{X}_t = \frac{1}{n}\sum_{j=1}^{n} X_{tj}. \qquad [3]$$

It is assumed that $X_{ij}'s$ are independently and identically normally distributed random variables with mean μ_0 and standard deviation σ_0.

The statistics V_t is standardized variable for variance as given in Quesenberry, (1995) which is

$$V_t = \Phi^{-1}\left[\mathrm{H}\left(\frac{(\mathrm{n}-1)\mathrm{S_t^2}}{\sigma_0}; \mathrm{n}-1\right)\right] \qquad [4]$$

where

$$S_t^2 = \frac{1}{n-1}\sum_{j=1}^{n}(X_{tj} - \bar{X}_t) \qquad [5]$$

be the t^{th} sample variance, and

$$\mathrm{H}\left(\frac{(\mathrm{n}-1)\mathrm{S_t^2}}{\sigma_0}; \mathrm{n}-1\right) = \mathrm{H(w;v)} = \mathrm{P(W \le w)} \;\; for\ W \sim \chi_v^2, \qquad [6]$$

the chi-square distribution with v degrees of freedom. D_t^2 has a chi-square distribution with 2 degrees of freedom when the process is in-control and therefore the

$$E(D_t^2) = 2. \qquad [7]$$

In this new scheme, the EWMA of the statistics of D_t^2 is obtained by

$$A_t = (1-\lambda_d)A_{t-1} + \lambda_D D_t^2 \qquad [8]$$

where $A_0 = E(D_t^2) = 2$ and $0 < \lambda_D < 1$. λ_d is a constant selected based on the shift Δ which is defined as

$$\Delta = \beta \times E(D_t^2). \qquad [9]$$

The optimal value for the constant λ_D differs based on the β to be detected quickly in a process. This new scheme issues an out-of-control signal if A_t is greater than the control limit H_A. The $H_A's$ for selected values of λ_D and ARLs were computed by simulation and displayed in Figure 2.1. The simulations were run until the standard error of the ARL was less than 1% of the pre-specified ARL.

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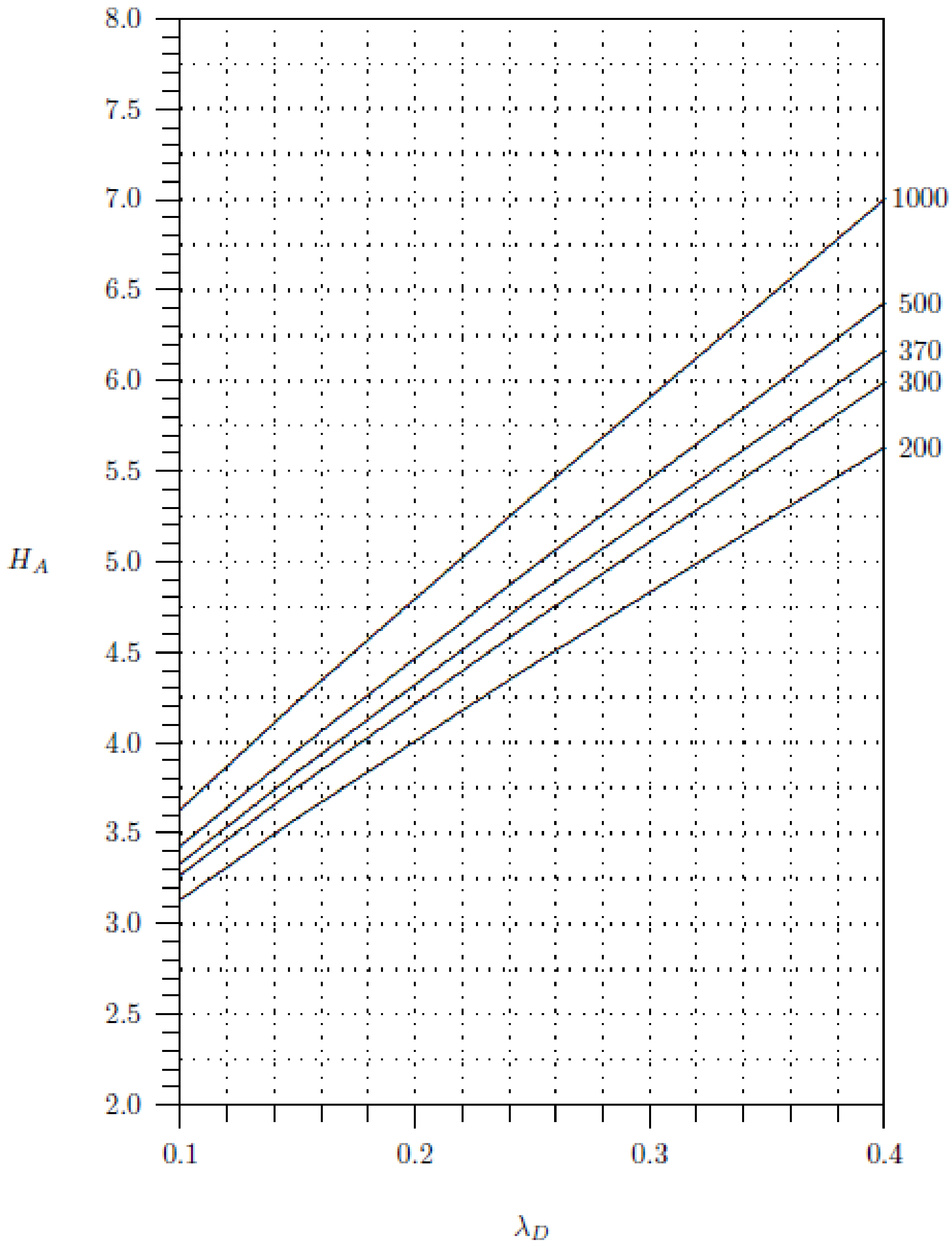

Figure 2.1. Control Limits for the λ_D values for the EWMAD2 Scheme with In-Control ARLs of 200, 300, 370, 500 and 1000.

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The optimal values of λ_D for various magnitude of the shifts in terms β has been found using simulations for different ARLs and displayed in figures 2.2 and 2.3. For a selected ARL and control limit, the λ_D value, that gives minimum out-of-control ARL for a given β value was found through simulation, and it has been selected as the optimum λ_D for detecting the β shift.

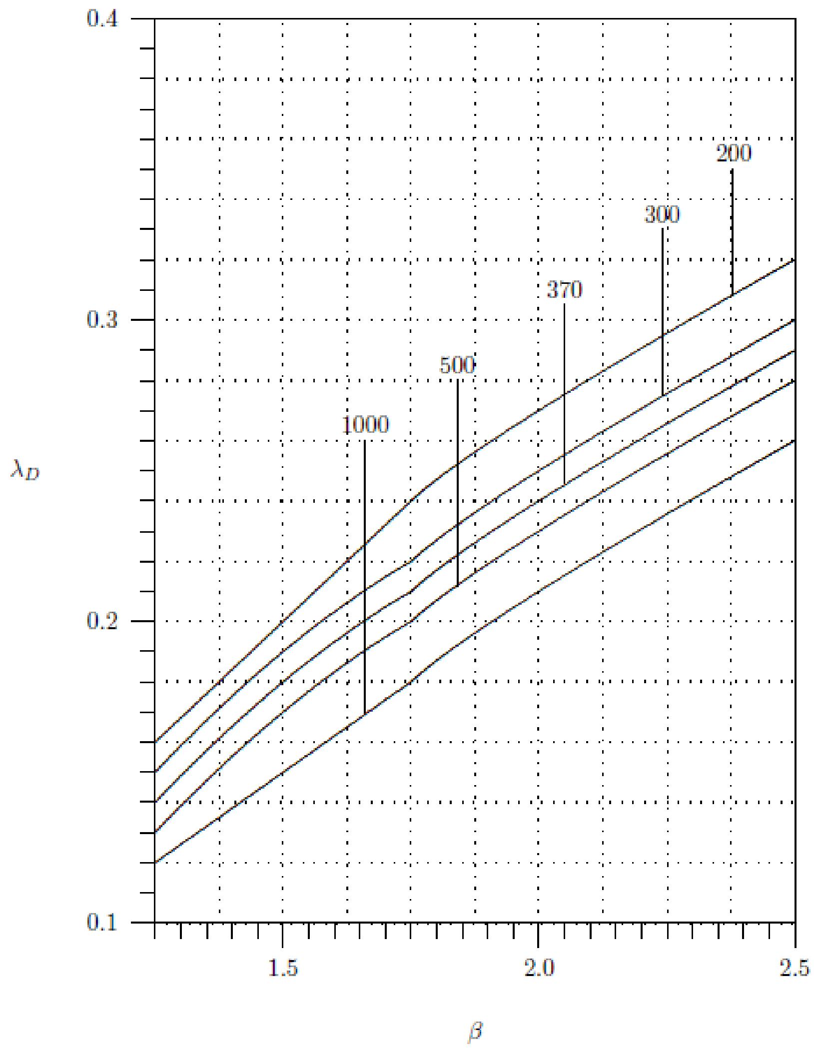

Figure 2.2. The Optimal of λ_D values for various magnitude of the shifts in terms β with In-Control ARLs of 200, 300, 370, 500 and 1000.

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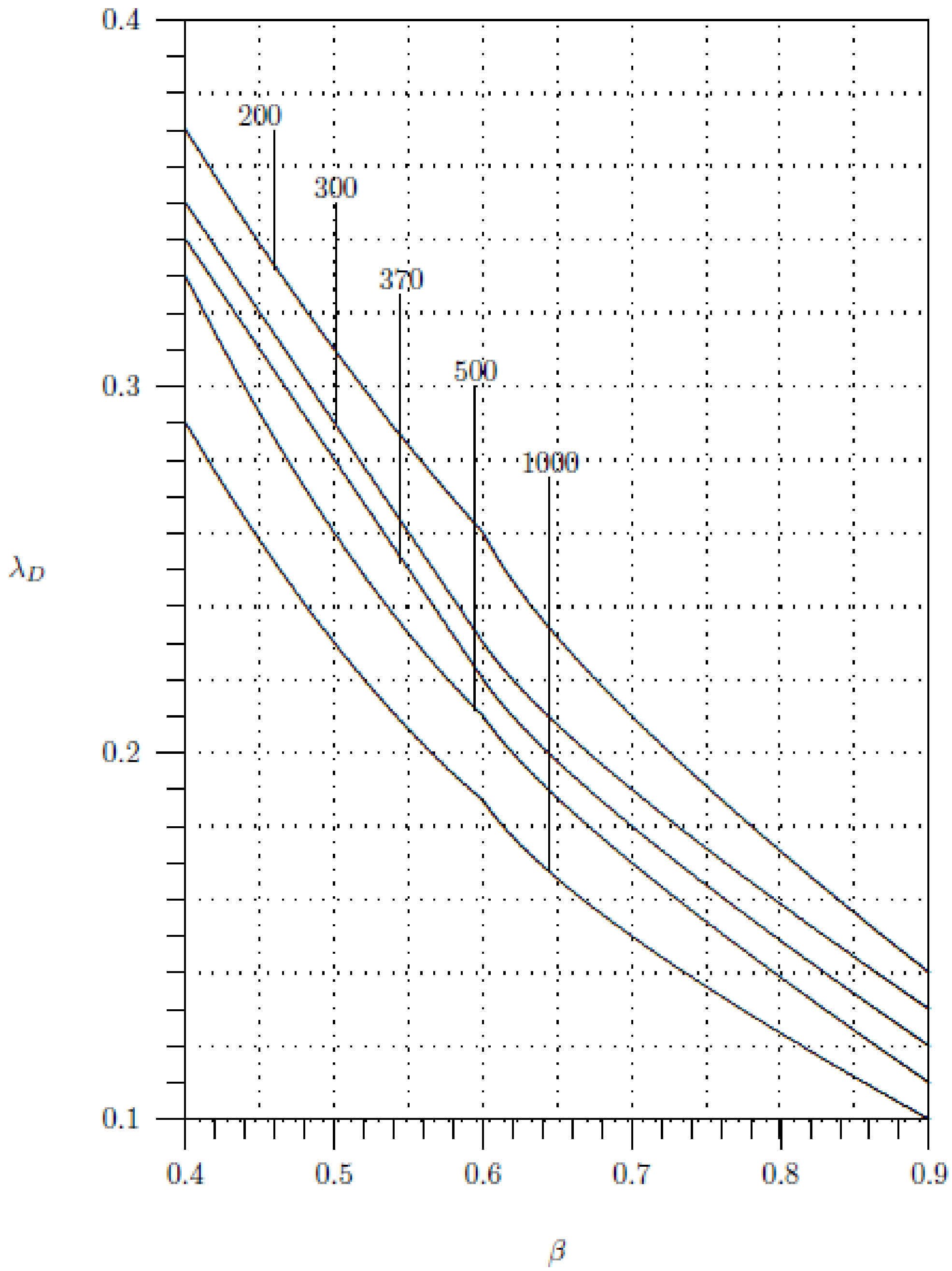

Figure 2.3. The Optimal of λ_D values for various magnitude of the shifts in terms β with In-Control ARLs of 200, 300, 370, 500 and 1000.

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3. DESIGN PROCEDURE

To design a EWMAD2 scheme, the following steps are recommended:

Step 1. Select the smallest acceptable in-control ARL of the scheme.
Step 2. Determine the magnitude of the shift Δ to be detected quickly.
Step 3. Select the λ_D that gives the minimum out-of-control ARL at the shift selected
from Figure 2.2 or 2.3.
Step 4. Given the value of λ_D from Step 3, find the control limit H_A which satisfies the in-control ARL specified in Step 1.

The EWMAD2 scheme can then be implemented by plotting the monitoring statistics A_t against the sample number t. A signal is issued if $A_t > H_A$.

4. AN EXAMPLE

Commonly known piston ring data in the quality control indusrty (available in https:// support.minitab.com/en-us/datasets/capability-data-sets/piston-ring-diameters/) is used to illustrate this new scheme. This data consists of 40 samples and each sample consists of measurements of diameters of 5 piston rings. The process mean and standard deviation for the piston ring data are estimated from the first 25 samples when the process is deemed in-control. The following steps can be adopted to implement a EWMAD2 scheme for the piston ring data if one wants to have an in-control ARL of 370 with minimum ARL when there is 1.5 times shift in D_t^2.

Step 1. Set the desired in-control ARL of the scheme to 370.
Step 2. The magnitude of the shift β to be detected quickly is 1.5 ($\Delta = 3$).
Step 3. The λ_D value that gives the minimum ARL at β = 1.5 is obtained from the Figure 2.2, which is 0.18.
Step 3. Since the optimal λ_D is 0.18, Figure 2.1 can be used to obtain the control limit H_A which is 4.1.

Then the EWMAD2 scheme can be implemented by plotting the monitoring statistics A_t against the sample number t and given in Figure 4.1. It shows that all the samples from sample number 37 onwards are out-of-control points.

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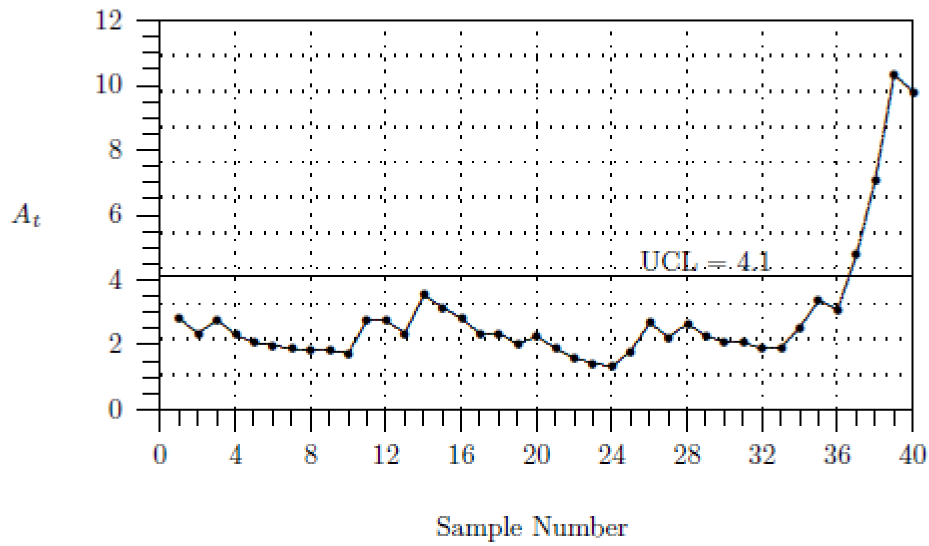

Figure 4.1 EWMAD2 Scheme for the Piston Ring Data.

5. Conclusion

In this EWMAD2 scheme, only the upper control limit H_A is needed because a change in the mean or variance will result in larger values of the monitoring statistics A_t. A problem with these schemes is that when a signal is issued, the chart does not indicate which process characteristic gives rise to the signal. Also the monitoring statistics indicate only the magnitude and not the direction of a shift. In process monitoring both magnitude and direction of a shift are important. Therefore, for these schemes, it is advisable to plot the monitoring statistics separately for each sample point. A big advantage of this new scheme is, unlike most existing JM schemes, control limits $H_A s$ and λ_D are not depend on the sample size and it has been discussed lengthily in Razmy (2016). This property gives fewer constraints in implementing this scheme in the industry.

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