$\tau_1\tau_2 - \delta$ SEMI CONNECTEDNESS IN BITOPOLOGICAL SPACES

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ABSTRACT : *In this paper, we are going to introduce some properties of $\tau_1\tau_2 - \delta$ semi connectedness in a bitopological space. Besides, we investigate several results in $\tau_1\tau_2 - \delta$ semi connectedness for subsets in bitopological spaces. In particular, we discuss the relationship related with δ semi connectedness between the topological spaces and bitopological space. That is, If a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2 - \delta$ semi connected, then the topological spaces (X, τ_1) and (X, τ_2) are δ –semi connected. In addition, we introduce the result which states that, a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2 - \delta$ semi connected if and only if X and ϕ are the only subsets of X which are $\tau_1\tau_2 - \delta$ semi clopen sets. Bitopological space does not exist for every metric space. But it exists only for special kind of metric spaces, called as "asymmetric metric spaces". There are many applications in various parts in mathematics.*

Keywords : Bitopological spaces, $\tau_1\tau_2 - \delta$ semi open sets, $\tau_1\tau_2 - \delta$ semi closed sets, $\tau_1\tau_2 - \delta$ semi connectedness.

1. INTRODUCTION

Kelly initiated the study about bitopological spaces in 1963. He introduced the concept "bitopological space" in his paper of London Mathematical Society in the year mentioned above. In addition, he established various properties in bitopological spaces, and got some generalized specific results. J.C.Kelly started his study about bitopological space from quesi-metric and its conjugate.

Semi open sets in bitopological spaces introduced by Maheswari and Prasad in 1977. Further properties were studied by Bose in 1981. Banerjee initiated the notion $\delta -$ open sets in bitopological spaces in 1987. Khedr introduced and studied about $\tau_1\tau_2 - \delta$ open sets. Later, Fukutake defined one kind of semi open sets and studied their properties in 1989. Recently, Edward and Balan introduced the concept $\tau_1\tau_2 - \delta$ semi open sets in bitopological spaces.

A quasi-pseudo-metric $p(\ ,\ )$ on a set X on the Cartesian product $X \times X$ satisfies the following three properties:

I. $p(x, x) = 0\ , \forall\ x \in X$
II. $p(x, z) \le p(x, y) + p(y, z),\ \forall\ x, y, z \in X$
III. $p(x, y) = 0$ iff $x = y\ , \forall\ x, y \in X$

However, $p(x, y) \neq p(y, x)$. Thus, quasi-metric cannot be a metric. But every metric is a quasi-metric.

For a nonempty set X, we define two topologies τ_1 and τ_2 on X. Then, (X, τ_1, τ_2) is called a bitopological space. Moreover, a topological space occurs for every metric space. But bitopological spaces occur for quasi metric spaces or asymmetric metric spaces.

Any subset A of a bitopological space (X, τ_1, τ_2) is called open, if A is both $\tau_1 -$ open and $\tau_2 -$ open. Throughout this paper, $\tau_i - int(A)$, $\tau_i -$

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$cl(A), \tau_i - \delta int(A)$, $\tau_i - \delta cl(A)$ and $\tau_i - \delta scl(A)$ be the interior, closure, δ −interior, δ −closure, $\delta -$ semi closure of A with respect to the topology τ_i respectively, $i = 1,2$. Let $\tau_j - \delta int(A)$ and $\tau_j - \delta cl(A)$ are the δ −interior and δ −closure of A with respect to the topology τ_j ; $j = 1s, 2s$. Here τ_s be the semi regularization of τ.

Now we introduce some basic definitions. Any subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2 - \delta$ semi clopen, if it is both $\tau_1\tau_2 - \delta$ semi open and $\tau_1\tau_2 - \delta$ semi closed. Any subset A of a bitopological space (X, τ_1, τ_2) is called τ_{12} −regular open, if $A = \tau_1 - int(\tau_2 - cl(A))$. Similarly, we can define $\tau_{21} -$ regular open set also. Any subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2 -$ semi open, if $A \subseteq \tau_2 - cl(\tau_1 - int(A))$. In a bitopological space (X, τ_1, τ_2), A is said to be $\tau_1 - \delta$ open, if for $x \in A$, $\exists$ $\tau_{12} -$ regular open set G s.t $x \in G \subset A$. Similarly, we can define $\tau_2 - \delta$ open set by substituting τ_{21} instead of τ_{12}. Moreover, Complement of $\tau_1 - \delta$ open set is called $\tau_1 - \delta$ closed set. Always $\tau_{1s} \subset \tau_1$ and $\tau_{2s} \subset \tau_2$; where τ_{1s} and τ_{2s} are the collection of all $\tau_1 - \delta$ open sets and $\tau_2 - \delta$ open sets respectively. Further, every $\tau_1 - \delta$ open set and $\tau_2 - \delta$ open set are $\tau_1 - \delta$ semi open set and $\tau_2 - \delta$ semi open set respectively. Recently, Edward and Balan established $\tau_1\tau_2 - \delta$ semi open sets in bitopological spaces. Any subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2 - \delta$ semi open, if $U \subseteq A \subseteq \tau_2 - cl(U)$, for some $\tau_1 - \delta$ open set U. Similarly, Any subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2 - \delta$ semi closed, if $F \supseteq A \supseteq \tau_2 - int(F)$, for some $\tau_1 - \delta$ closed set F.

2. METHODOLOGY

The objective of this paper is to introduce the concept "$\tau_1\tau_2 - \delta$ semi connectedness" in a bitopological space and discuss the relationship related with semi connectedness between topological space and bitopological space. First, we introduce the concept "$\tau_1\tau_2 - \delta$ semi connectedness" in a bitopological space. A subset Y is called a $\tau_1\tau_2 - \delta$ semi disconnected subset of a bitopological space (X, τ_1, τ_2), if there exist two $\tau_1\tau_2 - \delta$ semi open sets U and V such that $U \cap Y \neq \phi \neq V \cap Y$, $U \cap V \cap Y = \phi$ and $Y \subseteq U \cup V$. Otherwise Y is called a $\tau_1\tau_2 - \delta$ semi connected subset. Moreover, A bitopological space (X,τ_1, τ_2) is called $\tau_1\tau_2$ −δ semi connected space, if *X* cannot be expressed as the union of two non-empty disjoint sets *A* and *B* such that $\{A \cap \tau_1 - \delta scl(B)\} \cup \{\tau_2 - \delta scl(A) \cap B\} = \phi$. Suppose *X* can be so expressed, then *X* is called $\tau_1\tau_2 - \delta$ semi disconnected space and we write $X = A \backslash B$ and it is said to be $\tau_1\tau_2$ −δ semi separation of X. Now we introduce some examples:

Example(1): Let$X = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}, \{a, c, d\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{d\}, \{a, d\}\}$. Then, (X, τ_1, τ_2) is $\tau_1\tau_2 - \delta$ semi connected space.

Example(2): Let$X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{b, c\}\}$. Then, (X, τ_1, τ_2) is $\tau_1\tau_2 - \delta$ semi disconnected space.

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3. DISCUSSION AND RESULTS

First we shall show that the following result holds : Let U be a $\tau_1\tau_2-\delta$ semi connected subset of a bitopological space (X,τ_1,τ_2).Then if $U\subseteq V\subseteq \tau_2-\delta scl(U)$, then V is also a $\tau_1\tau_2-\delta$ semi connected subset. To prove this, we use the method of contradiction. Assume that V is not a $\tau_1\tau_2-\delta$ semi connected subset. Then, there exist two $\tau_1\tau_2-\delta$ semi open sets A and B such that $A\cap V\neq\phi\neq B\cap V$, $A\cap B\cap V=\phi$ and $V\subseteq A\cup B$. So $A\cap B\cap U=\phi$ and $U\subseteq A\cup B$, since U is $\tau_1\tau_2-\delta$ semi connected subset so at least $A\cap U=\phi$ or $B\cap U=\phi$, if $A\cap U=\phi$, then $U\subseteq A^c$. But $A\cap\tau_2-\delta scl(U)=\phi$ as A is $\tau_1\tau_2-\delta$ semi open. That is $A\cap V=\phi$ which contradicts to our assumption. Hence, V is $\tau_1\tau_2-\delta$ semi connected subset.

Next we introduce the following result : Let $\{U_\alpha\}_{\alpha\in I}$ be family of $\tau_1\tau_2-\delta$ semi connected subsets of a bitopological space (X,τ_1,τ_2) such that $\cap\{U_\alpha\}\neq\phi$, where $\alpha\in I$, then $\bigcup_{\alpha\in I}\{U_\alpha\}$ is also $\tau_1\tau_2-\delta$ semi connected. To prove this, let $x_0\in\cap\{U_\alpha\}$ and if possible $Y=\bigcup_{\alpha\in I}\{U_\alpha\}$ is not $\tau_1\tau_2-\delta$ semi connected, then there exist two $\tau_1\tau_2-\delta$ semi open sets U and V if $U\cap Y\neq\phi\neq V\cap Y$, $U\cap V\cap Y=\phi$ and $Y\subseteq U\cup V$, then let $x_0\in U$ (other case is similar). Now there exists $\alpha\in I$ such that $U_\alpha\cap V\neq\phi$ $also$ $U_\alpha\cap U\neq\phi$ (since $x_0\in U_\alpha$ and also $U\cap V\cap U_\alpha=\phi$ and $U_\alpha\subset U\cup V$ (since $U_\alpha\subset Y$) , which shows that U_α is $\tau_1\tau_2-\delta$ semi disconnected subset and it is a contradiction. So, $\bigcup_{\alpha\in I}\{U_\alpha\}$ is also $\tau_1\tau_2-\delta$ semi connected.

If a bitopological space (X,τ_1,τ_2) is $\tau_1\tau_2-\delta$ semi connected, then the topological spaces (X,τ_1) and (X,τ_2) are δ –semi connected. Since every $\tau_1-\delta$ open set and $\tau_2-\delta$ open set are $\tau_1-\delta$ semi open set and $\tau_2-\delta$ semi open set respectively, So if (X,τ_1) and (X,τ_2) are $\delta-$ semi disconnected spaces then the bitopological space (X,τ_1,τ_2) becomes δ –semi disconnected. But this is impossible. So (X,τ_1) and (X,τ_2) are $\delta-$ semi connected spaces.

A bitopological space (X,τ_1,τ_2) is $\tau_1\tau_2-\delta$ semi connected if and only if X and ϕ are the only subsets of X which are $\tau_1\tau_2-\delta$ semi clopen sets. To prove this, Consider a $\tau_1\tau_2-\delta$ semi connected space (X,τ_1,τ_2), let $\phi\neq A\neq X$ and A is $\tau_1\tau_2-\delta$ semi clopen set, then $X=A\cup(X\backslash A)$ is $\tau_1\tau_2-\delta$ semi disconnected in the bitopological space, which is contradiction. So X and ϕ are the only subsets of X which are both $\tau_1\tau_2-\delta$ semi clopen sets. Conversely, let X and ϕ are the only subsets of X which are both $\tau_1\tau_2-\delta$ semi clopen sets. If the bitopological space is $\tau_1\tau_2-\delta$ semi disconnected, so there exists a $\tau_1\tau_2-\delta$ semi disconnection $X=A\cup B$ of the bitopological space, So $A=X\backslash B$ and $B=X\backslash A$, then A and B both are $\tau_1\tau_2-\delta$ semi clopen sets and each of them is neither X nor ϕ. This is a contradiction. Therefore, (X,τ_1,τ_2) is $\tau_1\tau_2-\delta$ semi connected.

4. CONCLUSION

In this paper, Some results of $\tau_1\tau_2-\delta$ semi connectedness in bitopological spaces have been discussed. Furthermore, we have introduced $\tau_1\tau_2-\delta$ semi connectedness of a subset. We plan to extend our research work about uniform continuity in bitopological spaces. Further, we are interested to find some interesting results in bitopological spaces.

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